

DeMorgan's Laws

$$1 \quad (A+B)' = A' * B' \qquad 2 \quad (A*B)' = A' + B'$$

Prove one via t.t.

Fact about proving them via the axioms: Requires proof by contradiction, and is very hard.

Assume that it has been done, and that it doesn't use the fact that  $0'=1$ , then here are some easier theorems:

**Theorem:  $0' = 1$**

Proof:

$$\begin{aligned} (X')' + (X') &= 1 \quad \underline{\hspace{2cm}} \\ (X'*X)' &= 1 \quad \text{DeMorgan's Law} \\ (0)' &= 1 \quad \underline{\hspace{2cm}} \end{aligned}$$

**Theorem:  $A * A = A$**

Proof:

$$\begin{aligned} A &= A * 1 \quad \underline{\hspace{2cm}} \\ &= A * (A' + A) \quad \underline{\hspace{2cm}} \\ &= A * A' + A * A \quad \underline{\hspace{2cm}} \\ &= 0 + A * A \quad \underline{\hspace{2cm}} \\ &= A * A \quad \underline{\hspace{2cm}} \end{aligned}$$

**Theorem:  $A + A = A$**

$$\begin{aligned} A &= A + 0 \quad \underline{\hspace{2cm}} \\ &= A + (A * A') \quad \underline{\hspace{2cm}} \\ &= (A + A) * (A + A') \quad \underline{\hspace{2cm}} \\ &= (A + A) * 1 \quad \underline{\hspace{2cm}} \\ &= A + A \quad \underline{\hspace{2cm}} \end{aligned}$$

Challenge: Theorem:  $A + AB = A$  [For a Wednesday morning presentation?]

Definitions: **and, or, inverter, multiplexor (mux), decoder, encoder**

We are doing **combinational logic** (In which no state is remembered. Remembering state is called sequential logic.)