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How Has Christian Theology Furthered Mathematics?[1]

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Mathematica est inimicisima omnio theologiae.
[Mathematics is most unfriendly to all theology.]
—Martin Luther[2]

INTRODUCTION

This paper introduces possible candidates for case studies on the role of theology in mathematics. It is also a first attempt to develop criteria for the selection of candidates for my *Bibliography of Christianity and Mathematics*.[3] With material prior to the twentieth century, it is difficult to know what to include and what to exclude, since Christian presuppositions informed much scholarship in a vague, cultural way. I am looking for specific ways in which Christian theology has furthered mathematics. This paper introduces five examples from the Middle Ages and the nineteenth century wherein Christian theology potentially provided specific insights and motivations for the development of specific new branches of mathematics. I am interested in more substantial connections than merely being able to cite that Kurt Gödel calls the Church-Turing thesis of mathematical logic a "miracle," even though Gödel was a Lutheran who showed continued sensitivity to a Christian worldview.[4] However, an analysis and evaluation of the nature of the connections is beyond the descriptive scope of this paper.

I will not consider the converse question, "How has mathematics furthered theology?" because it has been treated by G.C. Henry in *Logos: Mathematics and Christian Theology*. He describes two ways in which mathematics has affected theology, both of them centered on geometry: Euclidean geometry was a model for Aquinas's *Summa Theologica* in its logical form; and non-Euclidean geometry was a model for relativity theory, which according to Henry in turn influenced process theology. At least in my first examples from the Middle Ages, the influence is clearly mutual, but I shall focus here on the influence of theology on mathematics.[5]

Another reason to avoid the converse question is that the answers to it have been trivial. For example, mathematics has been used to set the dates of Easter and of Saints' days. In medieval thought it was important to get those dates right, since your place in heaven was dependent on how many good works you did, including keeping the feast days. Such trivial examples would argue even for a relationship between Christianity and mathematics today because churches keep checking accounts! I am interested in theology's influences on the content of mathematics more than its influence on the methods and social conventions of doing mathematics.

I shall not discuss those who have done profound mathematics but who make no attempt to integrate their Christianity and their mathematics. These include Blaise Pascal, Leonhard Euler, August-Louis Cauchy, and others. One might think of Pascal's famous Wager as an exception to my claim in the case of Pascal. There he applied mathematical probability, of which he is one of the inventors, to the reasonableness of faith in God's existence. But in general Pascal does not integrate his mathematics and his faith. He "deliberately 'ties his hands' and refuses to look at any observation or experimental data bearing on the existence of a Christian God. He is writing for the man who will not countenance miracles, or the doctors of the church, or the witness of the faithful."[6] In any case, Pascal's Christianity does not shape his mathematics. That is the direction of influence that I want to address.

Since this is a descriptive paper, I shall assume as given each mathematician's view of mathematics. My own view is known as realism. By mathematics I mean broadly the "Science of Patterns."[7] That narrowly includes those fields that study the numerical (discrete) and spatial (continuous) aspects of reality.[8] I also include that part of logic which studies the patterns of the way in which concepts relate to each other.[9] Mathematics today is concerned about sentences which are universally true, not contingent on what the universe is like.[10] For an all too familiar example, $2 + 2 = 4$, where + may stand for "___ years grows by ___ years" or "___ quarts are mixed with ___ quarts." Likewise, to take an example from logic, *p and q* ⇔ *q and p*, where "⇔" means "are logically equivalent"; and "and" may stand for "furthermore" (an afterthought), "but" (a contrastive element is included), or "together with" (proximity). Such sentences are true precisely because they are not contingent. I like Bertrand Russell's observation on this point: Mathematics is "the subject in which we never know what we are talking about, nor whether what we are saying is true."[11]

Some people mistakenly take Russell's observation to mean that mathematics is a formalist's game with arbitrary rules. Nothing could be further from the truth, even today. The notion of formalism did not arise in mathematics until 1900 when David Hilbert proposed such a view.[12] More correctly, mathematics gets its power from the fact that the truth of its assertions is the truth about the properties of the nonmathematical world, not that world itself. Since the Middle Ages, mathematics has been concerned with accidents, not with substance.

I specifically exclude mechanism from mathematics. The attempts of natural philosophers to explain the world using mechanism are actually a step away from what I am attempting to do here. For example, in electronic circuit theory, an RLC circuit can be modeled by a mass-spring physical system, whether or not any mathematics is introduced. In turn, it is easier to explain mechanism in terms of mathematics. A contingent universe could have any kind of mathematics that applies, so an exercise in discovering how mechanistic theories, however mathematical, affect theology, although a valuable thing to do, is not my project. Again, Bertrand Russell says it clearly: "Physics is mathematical not because we know so much about the physical world, but because we know so little: it is only its mathematical properties that we can discover."[13]

As for theology, I should like to think of Christian theology, and not about theology in general. With Christian theology I mean beliefs about God held by Christians as Christian, whether referred to in the ecumenical creeds or not. Since William of Ockham is among my first examples, I quote Edith Sylla on his view, a view which I adopt in this paper. "Ockham is not at all shy about trimming the feathers of theology, reducing it from the full display of contemporary theological opinion to the minimum certified and authoritative Christian doctrines."[14] Since this is a descriptive paper, I shall assume as given each person's theological point of view. Ockham's nominalism would be viewed as heretical by Christian realists today; he viewed it as essential to defending the doctrine of transubstantiation. George Boole's dualism is the Manichean heresy against which St. Augustine argued; Boole viewed it as essential to understanding the problem of why there is evil. The theological assumptions these authors used are: God is one, God is infinite, God is triune, Christ is really present in the Eucharist. These assumptions are specifically Christian, even though only the last two are uniquely Christian.

What is the relationship, then, between mathematics as broadly conceived and Christian theology? How can mathematics and theology relate to each other at all? So far I have found the following three areas in which theology has affected mathematics. Here is the list, grouped by topic, to be introduced in the order shown below.

1. Algebra, freed from dependence on numbers, became the laws of thought itself;
   Nominalism by William of Ockham in the Middle Ages
   Quaternions by William Rowan Hamilton in 1843
   Boolean algebra by George Boole in 1849

2. Mathematical infinity was based on the nature of God Himself;
   in the Middle Ages and by Georg Cantor in 1883

3. Mathematical field theory replaced Newtonian particle theory.
   by William Hamilton and by James Clerk Maxwell

My examples cluster around the Middle Ages and the nineteenth century. Why has the concern of integrating mathematics and theology come in spurts historically? In the Middle Ages, Christianity was being confronted with secular ideas from Aristotle in particular. The Church faced the question, "How do Aristotle's well-thought-out physics, mathematics and metaphysics stack up against the Christian world view?" As Tertullian asked, "What has Athens to do with Jerusalem?" In the nineteenth century, there was a crisis in the foundations of mathematics precipitated by questions about the truth of Euclidean geometry and the validity of analytic methods, such as the calculus of Newton and Leibniz. George Berkeley, later to become Bishop, had already critiqued the calculus during Newton's lifetime, but calculus was so useful that it continued to be used despite Berkeley's objection. This nineteenth-century crisis in the foundations of mathematics—a need to make rigorous a growing body of informally argued mathematics—coincided with a general disillusionment with a sterile

Newtonianism.[15] Mathematicians wanted to shore up the foundations of their discipline. Those that were men of faith took confidence in the certainty of their faith to defend that mathematics from a uniquely Christian viewpoint.

The twentieth century faces a unique crisis—a loss of certainty both in faith and in mathematics. Davis and Hersh call mathematics today "fallible, correctable, and meaningful."[16] It makes sense in the light of this new view of mathematics called fallibilism to ask the question again: In what ways are mathematics and Christian theology related?

William of Ockham: Algebra Need Not Depend on Number Because Properties Need Not Inhere in Substances

The Franciscan William of Ockham (1285–1347) at Oxford University was at odds with the viewpoint of his predecessor Dominican Thomas Aquinas (1224–1274) at the University of Paris regarding the Real Presence of Christ in the Eucharist. I shall describe the changing view of algebra that Ockham promoted, and argue that his views on the Eucharist were a theological catalyst for his algebra.

The Church formally diverged from Aristotle in The Condemnation of March 7, 1277, issued by Bishop Étienne Tempier and the doctors of the Church. The Condemnation threatened excommunication for anyone who would teach the doctrines of Aristotle or of Aquinas. Pierre Duhem calls this the "birth certificate of modern physics."[17] It should be called the birth certificate of modern mathematics as well. These developments were inspired by Ockham whose philosophy was driven by his desire for a consistent theology.[18]

One doctrine condemned in 1277 was the proposition that God could not create a quality not inhering in a substance. Stated positively, the Church taught that God *can* create a quality *not* inhering in a substance. The bishop's motive was to preserve the doctrine of transubstantiation. In the Eucharist,

> according to Pope Innocent III and the Fourth Lateran Council, one has ...the substance of Christ accompanied by the accidents of the bread, but without the substance of the bread. Contrary to Aristotle, therefore, the accidents of the bread do not inhere in any substance—it would be absurd and irreverent to suppose that they inhere in Christ—and yet it is obvious to the senses that the qualities behave physically just as if the substance of the bread were still underlying them.[19]

How can this be? The ultimate answer of Aquinas is that the Real Presence of Christ is wholly supernatural and above reason. Ockham instead tries to rationalize. He argues that God could have made Christ to be present in the Eucharist even without the bread's accidents remaining visible. He assumes that God directly intervenes to prevent Christ from being seen.[20]

According to both Aquinas and Ockham, but not Aristotle, God can create accidents without substance, "although in the normal course of nature qualities always inhere in substances." Edith Sylla argues: "Were it not for the Eucharist, Ockham would have denied absolute reality to every accident. But the Christian faith teaches that sensible qualities such as color, taste and weight, remain *per se subsistentia* in the Eucharist without any subject." Further: "had it not been for the Eucharist, Ockham would have concluded that the only distinct *res* in the universe are substances."[21]

How does this relate to mathematics? Mathematics in classical Greek times was about the properties of objects. True, those objects might be triangles which exist in Plato's heaven, and not material objects, but mathematics did not study properties independent of objects. For example, to a classical Greek mathematician, x^1, x^2, and x^3 made sense because he could imagine spaces of 1, 2, and 3 dimensions; x^4 did not make sense, because there was no fourth spacial dimension. Mathematics today deals with abstract ideas, with attributes independent of substance. Ockham can be credited with the change: he wanted to be faithful to the doctrine of transubstantiation but to be consistent with observation. He argued, therefore, for a philosophy which allows the substance to change from bread to body, while the attributes (accidental properties) do not change. Ockham was able to save the nonappearance of Christ in the Eucharist![22]

Ockham's view freed mathematics from being about specific concrete objects, or even about specific objects in Plato's heaven.[23] It allowed mathematicians to concentrate on the form of an argument without regard to things that it was about. Although the analytic approach to doing mathematics goes back to the classical Greeks, in the Middle Ages mathematics was liberated from needing to be about something else. Mathematics was now the study of patterns of deductive reasoning about properties of objects in general.

A Commitment to Actual Infinity in the Mathematics of the Middle Ages Was Based on the Nature of God As Infinite

"Measure language," as mathematics and logic were together called in the Middle Ages, threatens to depersonalize us all. IQ tests as measures of intelligence are a specific example. More generally, mathematicians talk about the "values" of a variable as though the numerical scale were somehow measuring what is important for us to value. Initially computers were described in personal terms: "The machine is thinking." Now persons are described in computational terms: "I haven't input that data yet." The depersonalizing effect of mathematics and logic was recognized by the church in the fourteenth century. For example, the University of Paris Statutes of 1366 forbade the use of logic or mathematics as methods of arriving at theological truth.[24]

Why the fear of measure language in the fourteenth century? Murdoch suggests among other reasons that unanswered theological questions were catalysts: What is the "measure" of our love, or of our good works, or of God's infinity? Do angels move "continuously" from place to place? Can a person sin mortally "instantaneously" after being in a state of grace? Does free will, of God or of man, presuppose a "continuity" of choices?[25] Many fourteenth-century scholars faced these theological questions head-on using mathematics and logic, despite the condemnation of the church.

In turn, Christian theology clarified the mathematical notions of infinity and of continuity. Classical Greek mathematicians found paradoxes in assuming that actual infinities existed as completed totalities instead of as merely potential infinities.[26] Therefore, they rejected the notion of an actual infinity. For Christianity in the Middle Ages, however, the issue was not as simple. On the one hand, God was infinite, and space was "God's immensity,"[27] so there was an openness to the idea of infinity as a completed totality grounded in the nature of God. God Himself was the "urger of the infinite."[28] A mathematician might say that God is proof that actual infinity exists. On the other hand, Christianity rejected an ever-cyclic universe in favor of a created universe, so the concept that physical space may have existed for infinite time past was not well received. Also, Christians argued for a systematic accounting (counting!) at the last Judgment, so the number of souls could not be infinite.[29] Those who argued against actual infinity thought of God's nature as unapproachable by such weak tools as mathematics.

By the end of the Middle Ages, the existence of an actual mathematical infinity was a widely held view. The Condemnation of 1277, for example, influenced the debate in that direction. Bishop Tempier condemned as error this proposition: "God cannot make several worlds."[30] Since this proposition was Aristotle's basis for rejecting infinity as a completed totality, its rejection opened the door to permit an actual infinity.

The argument for an actual infinity had other support. It seems implicit already in Euclid's *Elements* of 300 B.C. There the ideas of continuity and of infinity are tied together, for if a line segment is continuous then it must have an infinity of points on it. That seems to be the inevitable result of Euclid's Proposition I.10, "Every line segment has a bisector." If you can always bisect the result of bisecting a segment, then you have an infinite number of points on the original segment. At Oxford's Merton College in the fourteenth century, Thomas Bradwardine (1290?–1349) argued just so, in a quite modern-sounding way in the context of a discussion in which continuity assumptions are independent of Euclid's postulates.[31]

So there is an infinity of points on a continuous line segment. But are they indivisibles? Henry of Harclay thought so, thereby rejecting the infinitesimally small. To refute Harclay, the Franciscan William of Alnwick distinguished between two propositions: (1) For all line segments there is a point such that it is the midpoint, versus (2) There exists a point such that for all line segments it is the midpoint. Alnwick claimed that the first is true; the second, false. Here is the first time that a logician had ever distinguished between what modern mathematicians call alternations of quantifiers.[32] It was soon applied to the discussion of infinity by an anonymous disciple of Ockham,[33] who considered the following two propositions: (1) For all finite numbers n there is a number greater than n; (2) There is a number greater than n for all finite numbers n. The first does not require a completed infinity to be true; if you give me n, I will give you $n+1$ as a greater number. The second requires a completed infinity.[34]

Alnwick appealed to God's "knowing" the successive midpoints of a line segment. A contemporary rendering of such an appeal might be, "In mathematics we think God's thoughts after him when we are not mistaken in our reasoning." To a mathematician today, existence in the mind of God is no different from simply existing. But the medieval argument is easily modernized. For "exists in the mind of God," read "is a possible world" in the quantum mechanical sense.[35] Then modal logic (the logic of possibilities) can be used to make Alnwick's argument acceptable to the secular mathematician in a way that shows that the mind of God was not superfluous in Alnwick's argument.

An actual infinity is paradoxical. For then either there is a whole equal to one of its proper parts (contra Euclid), or alternately one infinity is greater than another. Neither was acceptable to fourteenth-century thinkers. Nicole Oresme (1323?–1382) solved this dilemma by denying the applicability of measure language to infinity at all. Later Galileo and Newton were to do the same.[36] Mathematics, reasoned such thinkers, could apply to the profane, but not to God. In the nineteenth century, Cantor was to resolve the paradox of an actual infinity, as is discussed later in this paper.[37]

The most profound problem with infinity was already in focus in the fourteenth century: to come up with a set of axioms for handling

infinities. Peter John Olivi resolved the problem by using axioms for "directional equalities." Henry of Harclay was willing to modify Euclid's axiom, "The whole is greater than a proper part."[38] By the time the Augustinian hermit Gregory of Rimini gave lectures in Paris in 1344, Rimini had clarified the distinction between one set being contained in another, and one cardinal number being less than another: one set can be properly in another, but their sizes be equal.[39] Georg Cantor drew on these insights in the nineteenth century (see below).

If infinitesimals exist, they provide further support for the existence of an actual mathematical infinity.[40]

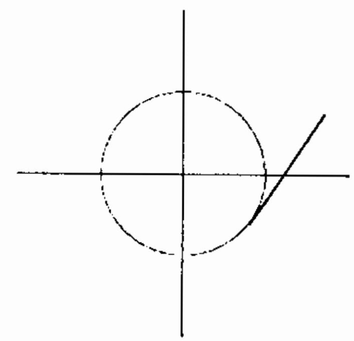

Figure 8.1: Horn Angle

Ironically, Oresme, who refused to apply measure language to the infinitely large, argued for the infinitely small on theological grounds. The angle between a curve and its tangent, a so-called horn angle, such as shown above, is infinitesimally small, he claimed. He argued first for a scale of perfections to preserve the *"ordo essentialis universi"* (essential order of the universe)[41]: There is an infinite difference, he claimed, between plants and animals, between animals and man, and between man and God.[42] By contrast, he reasoned, differences within species are as infinitesimal as the angle between two curves that are tangent to each other. Just how, in Oresme's mind, the theological grounds were linked to his acceptance of an infinitesimal small horn angle is a matter for further study. What is clear is that Oresme reasoned analogically rather than deductively at this point because he insisted on examples that were intuitive to him.[43]

In summary, three important mathematical theories were developed in connection with Christian theology in the Middle Ages. First, the recognition of quantifier alternations by William of Alnwick made an